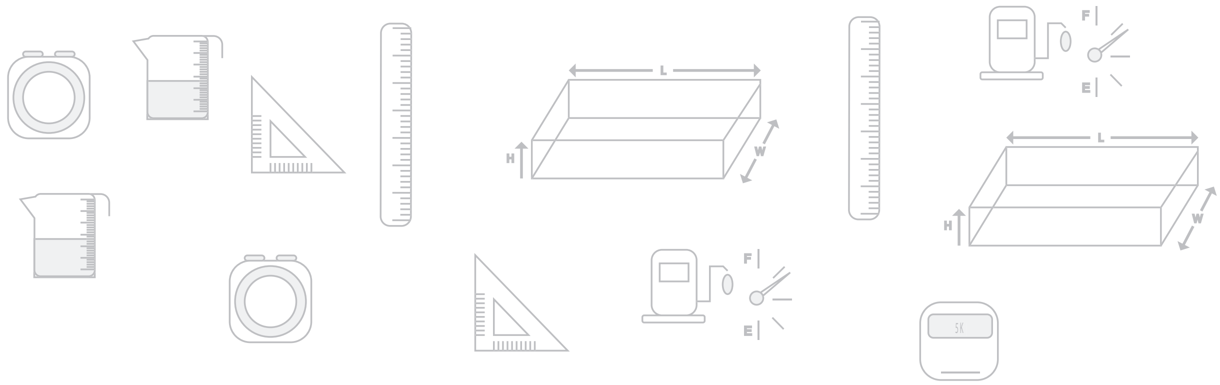
NUMERACY

Teaching maths in context



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Introduction

This book describes an approach to teaching mathematics based on applied or contextualised learning principles. The approach means that teachers teach mathematics from a contextual and task-based or investigative point of view, one where mathematics learning is developed by starting with an actual or modelled situation or task in which the mathematics is embedded. Investigations and projects are used as vehicles for learning. Teachers are encouraged to take into account students' interests and knowledge, and their own informal ways of doing mathematics, allowing the understandings and strategies learnt in and out of school to serve as valid resources.

Whilst primarily targeted at teachers teaching students who have become disengaged and/or disenfranchised from learning mathematics successfully during the middle to latter years of secondary schooling, this approach is also suitable for students who don't need formal mathematics as part of their further education, training or career aspirations. Often these groups of students overlap. Taking such an applied learning approach with an emphasis on problem solving can enable most students to succeed and progress, with students able to work from the same starting point but then proceed in different directions, learning and applying different mathematical skills to varying levels of sophistication. It can engage and motivate students in order for all of them to succeed in learning and applying mathematics to whatever level they are capable of—it does not have to be just for students who are considered to be unsuccessful at mathematics. It will support all students to become numerate and engaged citizens who are capable of using, transferring and adapting their mathematical knowledge and skills in this era of life-long learning and change.

This chapter provides a rationale for teaching mathematics in context, and explains what this means and what the potential benefits are for all students. The following chapters outline how teachers can develop, plan and teach in this way and provide models for this approach and ideas to use as a starting point.

Why do we need a different approach?

For some time now there has been evidence of dissatisfaction with what children are learning—or not learning—in mathematics classes, especially in the middle to latter years of secondary schooling. It appears there are too many students in these years of schooling who feel that mathematics is irrelevant to them, and have lost interest in studying or succeeding in mathematics.

For example, in Australia the introduction of the "New Basics" curriculum program in Queensland has typified some of the Australian-based issues and concerns about education and reform initiatives. As part of an initial longitudinal study (Lingard & Ladwig, 2001) into the quality of classroom learning (from Year 6 to Year 11) the study found that students learned only superficially and that students were mainly involved in trivial short-answer activities. This is supported by the results from the Trends in International Mathematics and Science Study (TIMSS) (Hollingsworth, Lokan & McCrae, 2003), an international study that included videos of mathematics classroom practice. As reported in this study, McIntosh describes a typical Australian Year 8 mathematics lesson:

> The teacher talks a lot, the students mainly reply with very few words, most of the time the students work, using only paper and pencil, on a repetitive set of low level problems, most presented via the board or textbooks or worksheets; discussion of solutions is mainly limited to giving the right answer or going through the one procedure taught. There is little or no opportunity for students to explain their thinking, to have a choice of solution methods or to realise that alternative solution methods are possible, and very few connections are drawn out between mathematical ideas, facts and procedures. (McIntosh, 2003, 108).

This is not confined to Australia. Similar TIMSS results are found in other western countries. In the US, Grouws and Cebulla (2000) reported that:

> data from the Third International Mathematics and Science Study (TIMSS) video study show that over 90% of mathematics class time in the United States eighth grade classrooms is spent practicing routine procedures, with the remainder of the time generally spent applying procedures in new situations. . . . . In contrast, students at the same grade level in typical Japanese classrooms spend approximately 40% of instructional time practicing routine procedures, 15% applying procedures in new situations, and 45% inventing new procedures and analyzing new situations. (Grouws & Cebulla, 2000, 17)

In 1990, as part of a national publication about US mathematics education, Davis, Maher, and Noddings (1990) described the situation as follows:

> By now nearly everyone has probably read, or at least heard of, the recent spate of reports showing that students in the United States are not doing very well in mathematics. . . . This leaves the United States with what might be called a war on two fronts. There is first, the fact of unsatisfactory results. But the second front is perhaps even more threatening: there is major disagreement on how to proceed in order to make things better. One school of thought would argue for "more" and "more explicit." That is to say, they would argue that the United States needs more days of school per year, or more hours of mathematics instruction per week, or more homework, or all of the above, together with a highly explicit identification of the knowledge that we want students to acquire, and a sharply directed emphasis on precisely this knowledge. . . .
>
> A different diagnosis and prescription might be said to tend in nearly the opposite direction. . . . These recommendations argue for making mathematics more natural, fitting it better into the context of children's lives, conceivably even moving toward less testing. (Davis, Maher, and Noddings, 1990, 1)

Nearly ten years later, Forman and Steen (1999) expressed similar sentiments:

> Despite mathematics' reputation as an ancient subject consisting of indisputable facts, mathematics education has recently become the source of passionate debate. At stake is nothing less than the fundamental nature of school mathematics: its content (what should be taught), pedagogy (how it should be taught), and assessment (what should be expected). . . . .
>
> At the risk of oversimplifying, this debate can be characterized as a clash between "traditionalists" who expect schools to provide the kind of well-focused mathematics curriculum that colleges have historically expected and "reformers" who espouse a broader curriculum that incorporates uses of technology, data analysis, and modern applications of mathematics. (Forman & Steen, 1999, 2)

What seems to be agreed from available evidence and research, is that many current and past methods of teaching mathematics to children have been less than completely successful. In secondary school classrooms in Australia and many other western countries, when teachers teach "maths", there is often a lot of "talk and chalk", students use a textbook and do lots of repetitive practice, they prepare for tests and exams, and they learn formal rules, often by rote.

There is little consideration of why and how the skills they are expected to learn can be put to use in the "real" world. The limitations of this style of mathematics instruction have now been acknowledged for some time in mathematics education, and a range of different strategies are recommended including those based on a constructivist view of mathematics (see below for a brief description of constructivism). However, it does still seem to be the main and traditional style of mathematics teaching in secondary schools.

How Australians are performing in mathematics

Is there evidence about Australians' abilities and performance in the learning and use and application of mathematics? Unfortunately, there is continuing and ongoing evidence from the early 21st Century that shows that both Australian adults' and school-aged students' success and performance in numeracy and mathematics is in need of improvement.

Australia has participated in a number of surveys of the literacy and numeracy proficiency of its adults, including in the Organisation for Economic Cooperation and Development (OECD) Programme for the International Assessment of Adult Competencies (PIAAC) survey, also known as the Survey of Adult Skills. There are five levels in PIAAC literacy and numeracy, from 'Level 1' to 'Level 5.' Owing to the relatively large numbers of adults at the lower end, PIAAC includes an additional 'Below Level 1.' In Australia, PIAAC was administered by the Australian Bureau of Statistics (ABS) with the first national and international results released in 2013 and an updated set of results, including for a second wave of countries, released by the OECD in 2016 (ABS, 2013; OECD, 2013b, 2016b).

According to the PIAAC survey, 22 per cent of Australian adults, which is about 3.5 million adults, are at the two lowest levels in numeracy (Below Level 1 and Level 1).This means that they can, at best, perform one-step or simple mathematical processes involving counting, sorting, basic arithmetic operations, simple percentages, and locating and identifying elements of simple or common graphical or spatial representations (ABS, 2013; OECD, 2013b, 2016b). Adults performing at Level 2 of PIAAC, where another third of Australian adults sit, can only identify and act on mathematical information and ideas embedded in a range of common contexts where the mathematical content is fairly explicit or visual with relatively few distractors. Tasks at this level tend to require the application of two or more steps or processes involving calculation with whole numbers and common decimals, percentages and fractions; simple measurement and spatial representation; estimation; and interpretation of relatively simple data and statistics in texts, tables and graphs (OECD, 2013b).

In contrast with the above numeracy results, the equivalent literacy percentages for PIAAC were considerably better—only 14% at Level 1 or below in literacy compared with 22% in numeracy, and 44% at Level 2 or below, compared with 54% in numeracy. Australia's numeracy achievement was just above the international mean, ranking 15th out of 34 countries, whereas Australia ranked 5th in literacy and scored significantly higher than the mean, showing that, relatively, the skills of adults are much weaker in numeracy than in literacy. In terms of gender, females performed significantly lower in numeracy, while the gender difference in literacy was much more balanced (ABS, 2013; OECD, 2016a).

This issue is illustrated further in the results of the OECD's Programme for International Student Assessment (PISA) which surveys a representative sample of 15-year-old students from a random sample of schools, every 3 years. In relation to mathematics, or what PISA defines and assesses as mathematical literacy, Australia's performance shows a clear and significant downward trend in its average mathematics score, from the 2003 survey administration to the 2015 survey (OECD, 2016a; Thomson et al, 2016). (For a comparison of PIAAC and PISA's numeracy and mathematical frameworks and assessments see Gal & Tout, 2014.) Similarly to the adult survey, the performance in numeracy has declined and our relative performance in mathematics is significantly lower than our performance in literacy. In PISA, 45% of Australian students do not reach the minimum level in mathematical literacy in PISA as determined by the Australian Curriculum, Assessment and Reporting Authority (ACARA) as the National Proficient Standard for PISA (Thomson et al, 2016). This level has been identified as the baseline because it represents 'a "challenging but reasonable" expectation of student achievement at a year level with students needing to demonstrate more than elementary skills expected at that year level' (ACARA, 2015, p. 5). This compares with reading literacy where only 39% of students did not reach the minimum level. In PISA 2015, Australia performed equal 12th in reading, significantly above the mean, but equal 20th in mathematics (OECD 2016a; Thomson et al, 2016).

The above research data and the fact that our relative performance in literacy is significantly better than in numeracy for both 15 year-olds and adults, is clear evidence that we need to improve Australia's performance in the teaching and learning of mathematics.

Connecting mathematics in the classroom with mathematics outside the classroom

Currently in many mathematics classrooms, especially in the middle years of schooling, the tendency is to use one approach to connect maths with the real world. That is to start by teaching the maths content and skills, followed by lots of practising of the maths skills and procedures. After this process, then and only then, might teachers get students to apply some of those skills into a real world context. And often this is done via maths 'word problems'.

The issue with many maths word problems is that they are often unrealistic, unauthentic, manufactured situations that actually make no sense in the real world. Here is an example found recently:

> A farmer has cows and chickens. He only sees 50 legs and 18 heads. How many are cows and how many are chickens?

This question makes no sense whatsoever—why wouldn't the farmer have simply counted the number of cows and chickens to start with—an easier task anyway? How and why would the farmer count the numbers of legs and heads? It is not at all a sensible or authentic scenario or question to pose. Why would a student want to engage with and answer such a nonsensical question?

Context based questions should be based on authentic stimuli or scenarios with questions asked being ones that someone would want answered. Such word problems as that shown above, disregard and challenge students' sense making and only continue to distance them from the real world, and the usefulness and value of mathematics. There is now a body of research about the issues with maths word problems (e.g., see Palm 2006, 2009; Verschaffel et al, 2009). As Verschaffel et al wrote when summarising some of the issues and challenges in relation to maths word problems:

> To summarise, there are a number of themes running through our collective work on this topic – past, present, and future – that apply, not just to the practices surrounding word problems, but to the teaching of mathematics in general:
>
> – We assume that the child learning mathematics (or anything else) comes with an innate drive for sense-making that should not be violated, and believe that attention should be given in teaching mathematics to make connections with children's lived experience."
>
> – Teachers of mathematics and others involved in managing this teaching – teachers of teachers, administrators, curriculum developers, test constructors – should not do so mindlessly, "because that is how it has always been done", rather need to reflect deeply about what they are doing, and why.
>
> – In contrast to typical mathematics teaching in schools, that arguably constitutes a training in simplistic forms of thinking – word problems being an extremely potent case in point – an understanding of the very idea that mathematics can be used to model aspects of reality, and that this process is complex, and has many limitations and dangers, is essential to effective and responsible citizenship. (Verschaffel et al, 2009, xxv)

An additional challenge is that authentic situations and scenarios that involve mathematical concepts, and their related stimuli and materials, are often complex. In relation to this complexity of mathematics in the real world, the Quantitative Skills in 21st Century Workplaces project undertaken jointly by the Australian Association of Mathematics Teachers (AAMT) and the Australian Industry Group (AIG) with funding by the Office of the Chief Scientist (2014), undertook research that identified and analysed the gaps between young peoples' numeracy skills and the expectations of 21st century workplaces. One of the more interesting conclusions of this project by the practising maths teachers involved is that the relationship between workplace mathematical skills and school mathematics could be described as 'distant' at best, and that although the skills observed appear to be fundamental, it is their use and application in work contexts that is not straightforward. The research found that workers perform sophisticated functions which require confidence to identify and apply mathematical skills in problem-solving situations and knowledge of the consequences of the procedures. The report found that workers need a blend of the following skills:

- ability to recognise and identify how and when mathematics is used in the workplace
- an understanding of mathematical concepts, procedures and skills

- an understanding of the kinds of practical tasks they need to perform
- the strategic processes they should be able to use in using and applying mathematics.

It was noted that current school teaching approaches generally emphasise these skills separately, and the connection between the world of the maths classroom and the world of work is very weak. One of the report's recommendations includes:

> Given that the transfer of mathematical skills to the workplace is not straightforward, there is a need to promote the teaching of mathematical skills and understandings in a way that encourages connections between mathematics and the real world and on the transfer of skills. The more contexts in which students are explicitly required and supported to transfer their mathematics, the more highly developed these skills will become. (AAMT & AIG, 2014, p. 4)

Summary

Given the above evidence, issues and factors, and that there are not strong connections between the world of maths within school mathematics classrooms and the world outside the classroom, a different approach to the traditional teaching of mathematics is needed if more students are to succeed in mathematics in the crucial years of schooling and to become numerate adults.

The following section makes connections with other areas of research into mathematics learning and teaching to demonstrate how an approach to teaching mathematics based on problem solving, applied or contextualised learning principles can help to achieve these aims.

Connections with other research

Other areas of research into mathematics knowledge, acquisition, learning and teaching have challenged the traditional approach to the teaching of mathematics described above. These include constructivism, ethno-mathematics, functional maths, numeracy education and gender issues.

Constructivism

As mentioned above, one of the major influences in mathematics education over the last few decades has been around alternatives to the traditional perspectives on what it means to learn and know mathematics, centred largely on the philosophy of constructivism as opposed to positivism (and variations and interpretations such as critical and social constructivism). This philosophy has helped mathematics educators reflect and think about what mathematical knowledge is and how it is acquired and what the implications are for teaching and learning (for example, see: Davis, Maher & Noddings, 1990; Ernest, 1998; Malone & Taylor, 1993).

Under a positivist philosophy, teachers are seen to act as the experts, and they transmit their knowledge directly to their students (a transmission model of teaching). Knowledge is seen as objective, and learning is about receiving the information handed down, absorbing the facts, and reproducing them. On the other hand, a constructivist philosophy sees knowledge as internal and subjective, where learning and understanding needs to be constructed by the learner, and where the individual learner makes sense of the mathematics. Teaching plays a facilitating and questioning role rather than a transmitting role. In social constructivism, learning is also seen as an activity where shared mathematical meanings are constructed with others and drawn from the learning environment. This also highlights the critical role and importance of language in the teaching and learning of mathematics – being able to talk about and share understandings of mathematical concepts and topics. Recent cognitive theories also hold that knowledge is constructed and restructured under a variety of constraints or conditions that either facilitate, or limit, what can be learned.

Some interpretations of the key implications of constructivism for classroom practice, adapted from Hatano (1996) are:

> 1. Mathematical knowledge is acquired by construction; therefore, students should be given the opportunity to actively participate in the learning process rather than be forced to swallow large amounts of information.
>
> 2. Cognitive restructuring is necessary to advance mathematical knowledge; to that end, instruction should induce successive restructurings of mathematical knowledge.
>
> 3. Mathematical knowledge is constrained by internal factors (cognitive, such as innate and early understandings and previous knowledge) and external factors (sociocultural, situated in contexts, such as peers, teachers, tools, and artifacts); it follows that each collection of factors may either facilitate or limit mathematical learning (Hatano, 1996, 211-213)

Ethno-mathematics

Often referred to as street maths, ethno-mathematics is about how mathematics is used in everyday, community and work situations outside the formal mathematics classroom and across different cultures (for example, see Gerdes, 1994; Nunes et al, 1993; Harris, 1991; Nelson, et al, 1993; Powell et al, 1997).

Bishop (1994) wrote that mathematics has generally been:

> "assumed to be culture-free and value-free knowledge; explanations of 'failure' and 'difficulty' in relation to school mathematics were sought either in terms of the learner's cognitive attributes or in terms of the quality of the teaching they received . . . 'social' and 'cultural' issues in mathematics education research were rarely considered" (Bishop, 1994, 15).

Ethno-mathematics has a number of lessons for mathematics education. For example, Zaslavsky (1994a) stated the following:

> Why is it important to introduce ethnomathematical perspectives into the mathematics curriculum? Students should recognize that mathematical practices and ideas arose out of the real needs and interests of human beings. . . .
>
> Students should learn how mathematics impacts on other subject areas—social studies, language arts, fine arts, science. Most important, they should have the opportunity to see the relevance of mathematics to their own lives and to their community, to research their own ethno-mathematics. (Zaslavsky, 1994a, 6)

Zaslavsky goes on to recommend how an ethno-mathematical perspective could be incorporated into school mathematics:

> Rather than a curriculum emphasizing hundreds of isolated skills, mathematics education will embody real-life applications in the form of projects based on themes and mathematical concepts. (Zaslavsky, 1994a, 7)

There are a number of messages to mathematics educators that emerge from ethno-mathematical research. One is the acknowledgment that formal, or school-based, mathematics is not the only mathematics—mathematical knowledge is acquired via both formal and informal learning, and that informal learning is as valuable as formal, school-based learning. Students should be encouraged to build on their "real life" mathematics experiences while also learning the conventions and practices of formal mathematics.

Functional mathematics

In the US, functional mathematics, where teaching and curriculum are connected to "real world" applications, is part of a related argument of how to improve school mathematics, especially at the secondary school level. Forman and Steen (1999) describe the need for a functional mathematics curriculum:

> Any mathematics curriculum designed on functional grounds … will emphasize authentic applications from everyday life and work. … By highlighting the rich mathematics embedded in everyday tasks, this approach … can dispel both minimalist views about the mathematics required for work and elitist views of academic mathematics as an area with little to learn from work-based problems.

> Neither traditional college-preparatory mathematics curricula nor the newer standards-inspired curricula were designed specifically to meet either the technical and problem solving needs of the contemporary workforce or the modern demands of active citizenship. (Forman and Steen, 1999, vi)

Forman and Steen then proceed to explain why and how such a functional mathematics curriculum could work to cater to both the traditional and reformist views of mathematics while at the same time making the learning of mathematics relevant and meaningful to all students.

Numeracy and mathematical literacy

The development and conceptualisation of numeracy has been another important influence on the teaching of mathematics (numeracy is a relatively recent and new term, first attributed to the UK Crowther Report in 1959, where numeracy was described as the mirror image of literacy (Crowther, 1959). The concept of numeracy is quite closely related to that of functional mathematics, where numeracy is often described as applying mathematics in a context. In Australia and internationally there has been ongoing discussion and debate in the adult education sector about defining the relationship between mathematics and numeracy and also to the concept of "critical" numeracy. Johnston (Johnston, 1994) has argued that numeracy in fact incorporates, or should incorporate, a critical aspect of using mathematics. She argues:

> To be numerate is more than being able to manipulate numbers, or even being able to 'succeed' in school or university mathematics. Numeracy is a critical awareness which builds bridges between mathematics and the real world, with all its diversity (Johnston, 1994, 34).

She continues:

> In this sense.... there is no particular 'level' of Mathematics associated with it: it is as important for an engineer to be numerate as it is for a primary school child, a parent, a car driver or a gardener. The different contexts will require different Mathematics to be activated and engaged in (Johnston, 1994, 34).

So numeracy here is described as making meaning of mathematics and sees mathematics as a tool to be used efficiently and critically for some social purpose. It is this broader view of numeracy that has driven much of the teaching, professional development and curriculum development in adult numeracy education in Australia since the 1990s. Examples of where it has been used in curriculum development are described below in the section *More to the point - does it work?*

The view of numeracy as making meaning of mathematics has also been used as a way of pushing for change in the teaching of adult basic mathematics in the USA (Schmitt, 2000; Tout & Schmitt, 2002). As Schmitt (2000) writes:

> Adult basic education and GED [General Educational Development] mathematics instruction should be less concerned with school mathematics and more concerned with the mathematical demands of the lived-in world: the demands that adults meet in their roles as workers, family members, and community members. Therefore we need to view this new term numeracy not as a synonym for mathematics but as a new discipline defined as the bridge that links mathematics and the real world (Schmitt, 2000, 4).

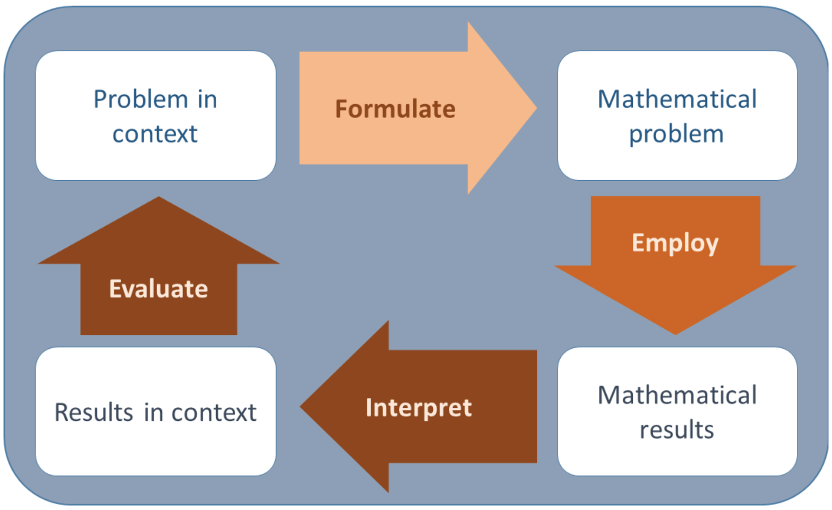
An international view of the concept of numeracy was behind the 2002 redesign of the 1992 International Adult Literacy Survey (IALS), an international large-scale comparative survey. The Adult Literacy and Lifeskills Survey (ALL) assessed the distribution of basic skills in the adult populations of participating countries and was designed to expand the domains assessed. It included for the first time an assessment of numeracy skills based on a similar broad view of numeracy (Gal, van Groenestijn, Manly, Schmitt, and Tout, 2003). This initial international survey of adults' numeracy skills was followed by the OECD's Programme for International Assessment of Adult Competencies (PIAAC) mentioned earlier which fundamentally kept to the same description and definition of numeracy (OECD, 2013a). The PIAAC definition of numeracy is:

> *Numeracy* is the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.

In the OECD's Programme for International Student Assessment (PISA) which surveys 15-year-old students in school, the definition of numeracy or what PISA calls mathematical literacy is:

> *Mathematical literacy* is an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013c)

Below is the diagrammatic conception of mathematical literacy developed for PISA 2012 (OECD, 2013) which is based around the need to assess students' capacity to transfer and apply their maths knowledge and skills to problems that originate outside school-based learning contexts. This is based around a mathematical modelling approach to mathematics education.



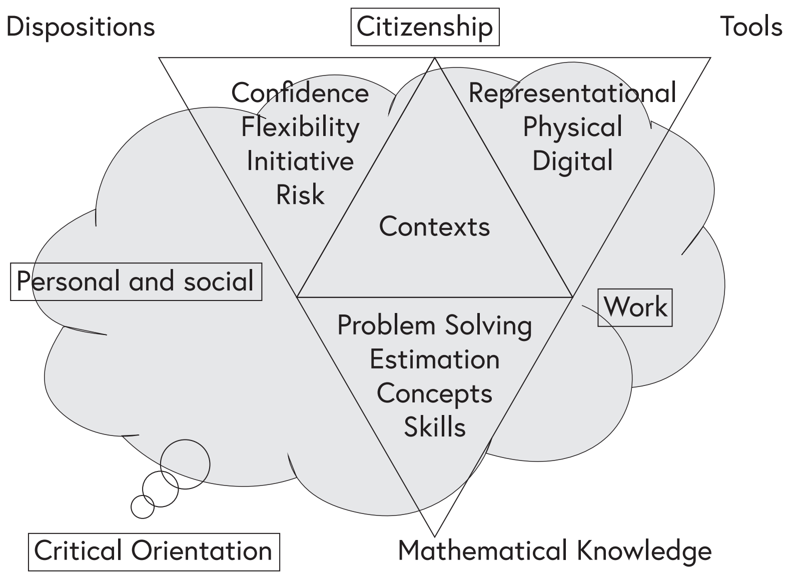
The processes outlined are key components of solving a real world problem, where the starting point is the problem in its context, not the maths.

- *Formulating* situations mathematically involves identifying how to apply and use mathematics to the real world problem—it includes being able to take a situation and transform it into a form amenable to mathematical treatment;
- *Employing* mathematical concepts, facts, procedures, and reasoning involves using mathematical concepts, procedures, facts and tools to derive a mathematical solution;
- *Interpreting and evaluating* the mathematical outcomes involves reflecting upon mathematical solutions or results and interpreting or evaluating them in the context of the initial problem.

This PISA process contrasts with the typical approach highlighted above of: teach some maths; practise some maths; apply some maths. The process in the real world requires a set of different skills undertaken in the reverse order—starting with the initial ability to identify the maths in the context and *formulate* it as a mathematical problem. Then, the second step is to *do* the maths, and *employ* skills and knowledge. Students then need to *interpret* and *evaluate* the outcomes of the maths and reflect on how the maths result(s) applies and fits in with the real world.

Both PISA and PIAAC describe mathematical literacy and numeracy as not synonymous with a minimal or a low level of mathematical knowledge and skills, but view the constructs as describing complex skills lying on a continuum.

Another more recent model for numeracy is shown in the figure below, and has some consistency with both the PISA model and PIAAC's framework. This model incorporates four dimensions of contexts, mathematical knowledge, tools, and dispositions that are embedded in a critical orientation to using mathematics (Goos et al, 2014, p. 84). These dimensions are described more fully in other publications (e.g., Geiger et al. 2014; Goos et al, 2014).



It is these broad views of numeracy or mathematical literacy as the bridge that links mathematics and the real world that lies behind the applied learning approach recommended in this book. Mathematics is an important tool in its own right, but there also needs to be a bridge that enables individuals to use mathematical skills and knowledge to solve problems embedded within social contexts and purposes.

School systems, especially in the primary school years, and increasingly in the middle years of schooling, have also been introducing numeracy programs to support their mathematics curriculum and teaching. This interest has again been partly to address concerns with the success of traditional mathematics teaching:

> There is also an increasing interest in numeracy, reflecting both a concern that Mathematics teaching is not succeeding, and also a desire to have a more relevant and context-related mathematics curriculum in schools. (Bishop, 2000)

Gender

Much has been written about gender and mathematics (for example, see Walkerdine, 1989; Willis, 1989; Harris 1997), and much of the research in this area is linked with the ethno-mathematics movement. A quote from a U.K. report by Harris (1997) demonstrates how these fields overlap in their view that informal, or "real life" mathematical knowledge is as valuable as that gained through formal instruction:

> Throughout the world it is women and girls who underachieve in mathematics. Mathematics is the study above all others that denotes the heights of intellect. Throughout the world, the activity that most clearly denotes the work of women, in both the unpaid, domestic sphere and in paid employment, is work with cloth. Work with cloth symbolizes women as empty-headed and trivial. Yet constructing cloth, decorating it during construction and converting it into garments, is work that cannot be done without involving spatial and numerical concepts that are the foundations of mathematics. (Harris, 1997, 191)

2 DESIGNING TEACHING PROGRAMS

Introduction

This chapter provides some ideas and ways to plan and design programs based on an applied or contextualised learning approach to teaching mathematics using an open-ended approach—utilising real materials and situations—making connections between maths and the real world. This includes brainstorming to identify themes and contexts and the starting questions/tasks; identifying the embedded mathematics that needs to be addressed; matching these against curriculum or standards; planning classroom activities and mathematics skills work; and also incorporating assessment and reporting requirements. As well there is some advice about what roles a teacher needs to take in the classroom in supporting students to learn mathematics this way, and what some of the skills are that students need to learn and be aware of.

How to plan

Teaching mathematics using an applied or investigative approach can be very demanding—certainly harder than teaching out of a textbook. It requires more planning and preparation in advance if it is going to work successfully. The advance planning helps in terms of:

- students having successful outcomes from their learning
- teachers effectively managing what is happening in the classroom
- teachers being able to successfully track and record the outcomes of their students
- teachers being able to meet curriculum outcomes.

On the other hand, given the more open-ended approach to the learning, there may be unexpected diversions—especially by more advanced and independent students. This is to be encouraged provided it contains relevant and valuable knowledge and mathematical content. The teacher needs to decide how to handle these excursions: How long might it take—can the class afford the time? Is it valuable? Could it be delayed until later?

The following flow chart maps out a process for negotiating and planning what to teach under an applied learning approach as described in Chapter 1. This is only a suggested process, and can be adapted to a particular context, curriculum or group of students.

Planning process cycle

1. Negotiate a theme or context	Find a theme or context of interest to the group of students: current popular topic, issue of concern or interest, or work.
2. Brainstorm	Investigate what the possible areas and topics exist that could arise out of the context/issue.
3. Identify starting question/task	Decide on the actual starting point and initial question(s) to initiate the student investigation or task—often the above stage will lead to too many options and pathways.
4. Identify mathematics content	Investigate the mathematics that is embedded within the chosen topic and theme and the particular starting question. Refine question/task if needed.
5. Map against curriculum	Map the mathematics identified against any curriculum outcomes or descriptions that have to be met. Identify any gaps and refine question/task if needed.
6. Identify resources needed	Based on the above stages, identify the resources needed for the students to undertake the task/investigation and for teaching the mathematics content and skills.
7. Plan assessment	Include decisions about how the students will be assessed and how you will collect evidence—needs to be ongoing, formative assessment not just summative assessment at the end.
8. Do it—teach/facilitate	Students undertake the investigation—teacher is mainly a facilitator and resource person. But intervention to teach the mathematics and problem solving skills are necessary.

3 MAKING MEANING

Introduction

A key skill in modern society relates to interpreting and reflecting on numerical and statistical information in various documents, texts and other media. This requires people to be able to view, read, understand and interpret such information no matter where it is presented. It is crucial in many spheres of life—at home, at work, in the community and certainly personally. Documents and texts these days can be in newspapers and magazines; on the internet; on social media, on TV; in information and leaflets from councils, banks, government agencies, utilities such as electricity, telephone companies and community organisations; in information printed on goods purchased from different sources; and so the list continues.

To NOT understand the numerical and statistical information and explicit and implicit messages embedded within such texts prevents people from engaging with, and participating fully, in society. Teaching about reading, understanding and interpreting such numerical, quantitative and qualitative information is an important task that supports students to become numerate and empowered citizens. In some cases it may be critical information (e.g., about health or safety issues) or it may have financial consequences (e.g., about fines, overdue bills or gambling debts) or it simply may be a matter of enjoyment and engagement (e.g., reading a newspaper article of interest or about sports results).

Types of mathematical information embedded in texts

Most of the numerical and statistical information in reading and interpreting such texts and information will be around these main types of mathematical information:

- number
- statistics and data analysis
- chance and probability
- maps, plans and diagrams.

Some may require understanding of:

- pattern, relationship and algebra.

Mathematical information may be encoded in different ways such as:

- a diagram or chart
- graphs and tables may be used to display statistical or quantitative information
- a map, plan or diagram of a real entity (e.g., map of a city, diagram of a piece of equipment, a house plan)
- various types of texts with numerical or quantitative information (such as in a news item, reports, public information, bank forms, etc.).

Numerical information in texts

Two different kinds of numerical information in texts may be encountered. The first involves numerical and statistical information represented in words or phrases that carry mathematical meaning. Examples are the use of number words (e.g., "three" instead of "3"), basic mathematical terms (e.g., fraction, percent, average, proportion), or more complex phrases (e.g., "crime rate increased by almost a quarter") which require interpretation.

The second is where the information is expressed in notations or symbols (e.g., \$3.27b, ± 10, 24.5%, 450 m^2, etc.), but is surrounded by text that despite its non-mathematical nature also has to be interpreted in order to be understood.

Between the lines—reading, interpreting and more

But not only is it important to be able to read and understand the numerical and mathematical information embedded in the text, readers may also need to make a judgment about how the information actually applies to the situation or context, or whether the data is being presented fairly or not. This interpretation can also incorporate a critical numeracy aspect, where the purpose of the task, the validity of the data or information presented, or the meaning, interpretation and implications of the results, may need to be questioned.

The implication for classroom practice is that mathematics or numeracy teachers need to be aware of how crucial the mathematical knowledge embedded within materials and information is, and how vital it is to understanding the messages and information. Teachers need to therefore use examples of such materials in their classrooms to demonstrate how the mathematics is embedded and making explicit to students what role the mathematics plays in both presenting the information in the first place, and then secondly in being able to assess and critically review the information and any explicit or implicit meanings. In many instances the information will be quantitative and statistical and require mathematical knowledge related to number, chance, data and statistics.

Mathematics areas covered

In the table below, there is a list of possible topics related to reading and interpreting information that could be considered as areas for investigation, and therefore for teaching a range of mathematics skills.

Topics around the context of interpreting information

Topic and context	Areas of mathematics
Medicine and drugs • Medication labels and instructions • Articles and leaflets about drug usage, including alcohol and smoking • Websites about drug use	Number & algebra Statistics & probability
General news articles and information, including daily news, current affairs, etc. • Newspaper and magazine articles • Websites on the Internet, including Facebook • TV shows including news and current affairs	Number & algebra Statistics & probability
Music and entertainment • Newspaper and magazine articles • Radio shows and their websites • Music and band websites on the Internet • TV shows and their websites	Number & algebra Statistics & probability

Sports articles and information • Newspaper and magazine articles • Sports bodies, competitions and shows on the Internet • TV and Radio shows	Number & algebra Statistics & probability Measurement & geometry
Advertising materials • For buying goods and services • Best buys • Advertisements in newspapers, magazines and the Internet	Number & algebra Statistics & probability
Financial information • Brochures and leaflets from banks, utilities, government • Bills: utilities such as phones, electricity, etc. • Bank statements and details including via the Internet • Credit card payments and personal loans and finances	Number & algebra Statistics & probability
Application forms including tests • Learner drivers permit and driver's licence • Job application forms • Youth allowance application • Concession card application	Number & algebra Statistics & probability
Gambling • Popular forms of gambling: Tattslotto and the rest • Casinos and gambling: including financial implications for individuals and society • Sport related gambling: horse racing, other sports betting, etc	Number & algebra Statistics & probability
Workplace documents and information • Employment forms • Standard operating procedures, instructions • OH&S guidelines • Safety warnings • Chemicals and dangerous goods • Quality control procedures and processes.	Number & algebra Statistics & probability Measurement & geometry
Home hazards and safety information • Leaflets on goods or their packaging eg. electrical equipment, assembly instructions • Chemicals and sprays	Number & algebra Statistics & probability Measurement & geometry
Statistical data and information • Data published about national, regional and local data e.g.: • census data • data from research into car accidents, drug use, etc. • election results and opinion polls	Number & algebra Statistics & probability

Weather information • Newspapers • The Internet • TV or radio.	Number & algebra Statistics & probability Measurement & geometry

The area of interpreting and making meaning from numerical and statistical information included in texts can address many of the different types of mathematical knowledge and capabilities described in Chapter 1.

Different types of mathematical knowledge and capabilities covered

Type of knowledge or capability	Associated capabilities	Covered here?
1. Utilitarian knowledge	To be able to demonstrate useful mathematical and numeracy skills adequate for successful general employment and functioning in society.	Yes—reading, understanding and interpreting numerical and statistical information is a crucial and practical skill in many spheres of life—work, community and personally.
2. Practical, work-related knowledge	To be able to solve practical problems with mathematics, especially industry and work centred problems.	Yes—would need to illustrate this through some vocational or workplace examples such as Standard Operating Procedures, OH&S instructions or quality control processes.
3. Advanced specialist knowledge	To have an understanding and capabilities in advanced mathematics, with specialist knowledge beyond standard school mathematics (including advanced high school specialist study of mathematics to knowledge of university and research mathematics).	Maybe yes, depending on whether the students are interested in examples and situations where higher level skills are required for interpreting and analysing the embedded maths. Examples could be found in higher level statistical analysis required, for example, in quality control processes using quality control charts based on upper and lower control units.
4. Appreciation of mathematics	To have an appreciation of mathematics as a discipline including its structure, subspecialisms, the history of mathematics and the role of mathematics in culture and society in general.	Yes. Demonstrating how numerical and statistical information is a crucial and practical skill in many spheres of life and that there is vital mathematical knowledge and understanding behind this skill, should show students the significant role mathematics plays in culture and society in general.
5. Mathematical confidence	To be confident in one's personal knowledge of mathematics, to be able to see mathematical connections and solve mathematical problems, and to be able to acquire new knowledge and skills when needed.	Yes. Again, given the crucial and practical nature of understanding numerical and statistical information, working with students to realise this and make the connections between the mathematics and the different texts they will meet should provide a good basis for building their confidence and capacity.

6. Social empowerment through mathematics	To be empowered through knowledge of mathematics as a highly numerate critical citizen in society, able to use this knowledge in social and political realms of activity.	Yes. This area of knowledge is almost the main purpose of studying this topic - interpreting and reflecting on numerical and statistical information in documents and texts - is because it supports students to become more numerate and empowered citizens.

In this area of making meaning of information, Category 6, Social empowerment through mathematics, is a primary purpose and outcome of analysing how information is used and embedded in different texts and materials. It is important to choose as examples texts and materials that are:

- topical
- of particular interest to the group of students, or
- of general and popular interest.

There will always be current topics and issues that enable teachers to address the issue of making meaning of information, some of which are listed in the table above. Combined with the knowledge about the different types of mathematical knowledge described in the table above, and wanting to cover as many of these as possible, teachers will be able to use the planning strategies outlined in Chapter 2, to develop a number of learning activities and investigations around making meaning of information.

The following examples illustrate how texts can be used to analyse the information and address the mathematical content. While the source of the material is of a particular socio-cultural or political context, various situations of a like type can be readily identified in local materials and of interest to the cohort of students. The suitability of material to be used and analysed goes hand-in-hand with local teacher knowledge—both of the local contexts but also of the student interests.

Thus, while the sample materials in Figures 1 and 2 are drawn from the Australian context, the issues are ones that could be easily matched and used in other contexts. The issues of child drug use and gambling (Figure 1) are relevant to both more developed and developing nations. Similarly in Figure 2, sports information whether on the internet, or on TV or in newspapers, is of international interest in sports such as football (soccer), basketball, netball, cricket, hockey, rugby and other sports, as well as national codes of sport like Australian Rules Football.

As mentioned earlier, teachers need to use examples of texts such as these to demonstrate how mathematics is embedded and make explicit to students what role the mathematics plays in both presenting the information in the first place, and then secondly in being able to critically review and assess the information for any explicit or implicit meanings.

5 MAKING ALGEBRA WORK

Introduction

One of the main areas of frustration and negative attitude for a lot of secondary school mathematics students is that of engaging with the abstract world of algebra—usually introduced in the first year of secondary school. This attitude is often then carried into adulthood.

Students often have a lot of difficulty understanding and using algebra—they don't see its relevance or use, and are unable to come to grips with its foundations in generalisation, pattern and relationships, and ultimately its abstraction (and beauty to a lot of teachers and successful mathematics students). Yet algebra forms the basis and subsequent content of much of the senior secondary school curriculum, and failure to learn how to understand, use and apply algebraic skills effectively is a barrier to many students' future success and continuation with mathematics.

Algebra has many uses and benefits, and these need to be highlighted and used to motivate and engage students to learn about it. Algebra can be seen as a way of making and applying generalisations and developing generalisable expressions; it can be seen as being about relations and functions and about formalising patterns in number; and algebraic understanding underpins the use and application of formulas in a wide range of workplace situations such as those related to measurement.

Teachers themselves need to be able to see the usefulness of algebra and how it is used in this variety of ways, and need to be able to demonstrate some everyday application and relevance which may assist students to see that there is some way of connecting with the world of algebra and seeing how and why it works the way it does, and that there is a purpose behind learning about algebra. Similarly teachers need to be able to present to students arguments about the use and conventions and rules of creating, writing and manipulating algebraic expressions and formulae.

However, teachers and mathematics educators often say, for example: "Learn this now: you will use it later in more advanced mathematics" (Gough, 2005) to justify why students need to learn formal, abstract mathematics in secondary school—not a very compelling argument to the struggling mathematics student who has no need, intention or interest in continuing to study formal mathematics. It is an example of the 'just in case', rather than the 'just in time' approach to education. For many students they are studying formal mathematics 'just in case' they might need it later, whereas if the more formal and abstract aspects of mathematics (in this case, algebra) can be connected to contexts, uses and applications from students' current or potential future interests then the 'just in time' motivation may be able to be utilised to engage students with learning algebra.

The aim of this chapter is to look at a few of the ways of giving meaning to, and understanding, algebra and using formulas. It will suggest a way of how algebra can be developed and illustrated through generalisations and abstractions of relevant, real life situations, and show how there is a need for formulas and algebra in real life, work based situations. Hopefully, this is a way in which disengaged students can see a purpose for learning and using algebra and to therefore engage more fully with it in their school lives and work.

As argued in Chapter 1, many occupations and trades use a wide range of mathematical skills everyday—shop keepers, builders, electricians, plumbers, dressmakers, chefs, carpenters, mechanics, gardeners and landscapers, sportspeople, drivers, bookkeepers, asphalters and pavers, and so on. In many of these occupations there is the need to engage with the understanding and application of algebra and formulae.

A common approach to teaching algebra at school

It may be useful for teachers to reflect on the following scenario.

What are the most common experiences of the teaching and learning algebra at school, especially in the middle and upper years of secondary school? How is it introduced? How is it explained? How is it practised? Reflect on how algebra is taught: think about questions such as:

- Was algebra made meaningful or relevant to the students? Was algebra related to everyday or real situations?
- What methods were used to teach it?
- Were there any hands on materials used, or was it predominantly taught by the teacher at the blackboard or out of textbooks?
- How, or why, do teachers argue for the value and importance of learning algebra?
- Did it cause students to feel worried or anxious about maths, or turn them off maths?

What do your responses to these questions mean for the current practices and experiences in your school—do you think there is a problem with how algebra is taught and with how students engage with it?

Often algebra is seen and presented as a set of abstractions and rules that need to be followed, with page after page of repetitive practice exercises involving x's and y's. Many students never see the logic and patterns within these tasks—and rather than improving understanding, these approaches can lead to more confusion and disengagement (e.g., Hart, 1981, p. 212).

Introducing and connecting with algebra

Algebra can be used to generalise situations where patterns or repeated calculations or situations occur and while this is not what all of algebra is about, it is a very essential part of it, and presents a crucial way in to engaging and motivating students to understand the workings and use of algebra. And this can happen in lots of different everyday situations. For example, how you:

- work out costs (e.g. how much you pay a service person, how much you pay for amounts of materials or goods, etc)
- work out sports scores
- work out how long it might take you to travel somewhere, and how much you might be paid if you get travel allowances
- work out areas and volumes to calculate the amount of mulch, soil or sand, asphalt etc needed for a job
- convert between mass and volume if you know the relative density of the substance (e.g. asphalt)
- calculate interest
- generalise when calculating or estimating almost anything that has a regular pattern or cost, production, usage, etc.

It is often the case that young people and adults don't realise that they are using a form of algebra to solve a problem or work something out—and this is at least partly because these connections were not made when they initially engaged with and tried to learn about algebra in school. This inability to see the relevance and use of algebra, even just in terms of its use in generalising, often stops students from engaging with mathematics in the middle years of schooling. However, algebra can be shown to be connected to and embedded in many areas of life, work and study.

Types of relationships and terminology

One of the benefits of putting algebra into context is that it then provides a vehicle for introducing and talking about the simplifications, symbols (or lack of the use of symbols as with multiplication), conventions, terminology and types of relationships that are used and how they are represented formally. For example, if the formula being discussed makes sense in the real world (as in the Australian Rules football example demonstrated later on) then terms and their meanings such as variable, dependent and independent variables, constants, etc. can make sense to students—it often explains words and meanings that have seemed mysterious previously. They can make everyday sense and connect to student's experiences and understandings.

This approach enables students to realise that algebra can be useful and can be based on real-life situations—it isn't just an abstract game played within school maths classrooms and that they need to grapple with and understand for no particular reason. The issue then is of extending that to explain how mathematics works as an independent and abstract system. How to teach the structure, conventions and manipulations of algebra is an important and vital aspect of teaching and learning mathematics. However, the aim behind this chapter is to engage students to get them to see that there can be an underlying purpose, meaning and relevance to using algebra which can be a stepping stone to accepting and understanding the more formal rules and abstractions of mathematics.

Mathematics areas covered

In the table below, there is a list of possible topics or work situations that are related to algebra and formulas that could be considered as areas for investigation.

Topics around algebra

Topic	**Algebra and mathematics content**
Australian Rules Football and other sports	• Rates • Algebra, formulas and relationships • Extension—simultaneous equations and graphs
Service charges	• Rates • Algebra, formulas and relationships • Extension—simultaneous equations and graphs
Nursing	• Measurement • Rates • Algebra, formulas and relationships • Direct and inverse relationships
Asphalting and paving	• Measurement • Rates including speed, spray rates, etc. • Algebra, formulas and relationships • Areas and volumes, including triangles, circles and compound shapes • Density (volume versus mass)